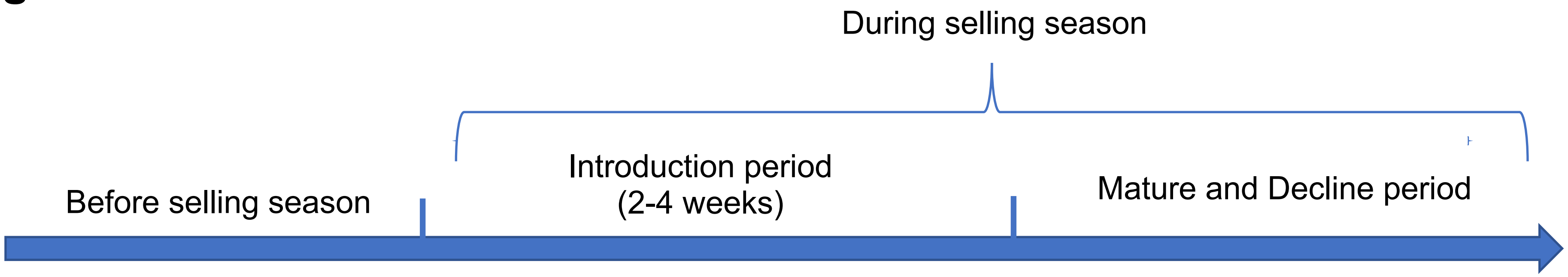
Reorder Strategy under Precision Evolution and Capacity Constraint.

Background



Order without data
Inaccurate !

Product Classification:

Important method to balance the demand and supply

- Fast-selling – reorder and sell the product in more stores
- Slow-selling – promotions
- Current Classification Method :
  - Manual threshold based method (Time-consuming & Error-prone)
  - 1000+ SKU

Reorder Strategy :

Problem: when to reorder? Which product to be reordered?

- Prediction can be wrong. The precision rate increases with the days on market.(**Precision evolution**)
- Products are launched gradually over time.(**Capacity constraint**)

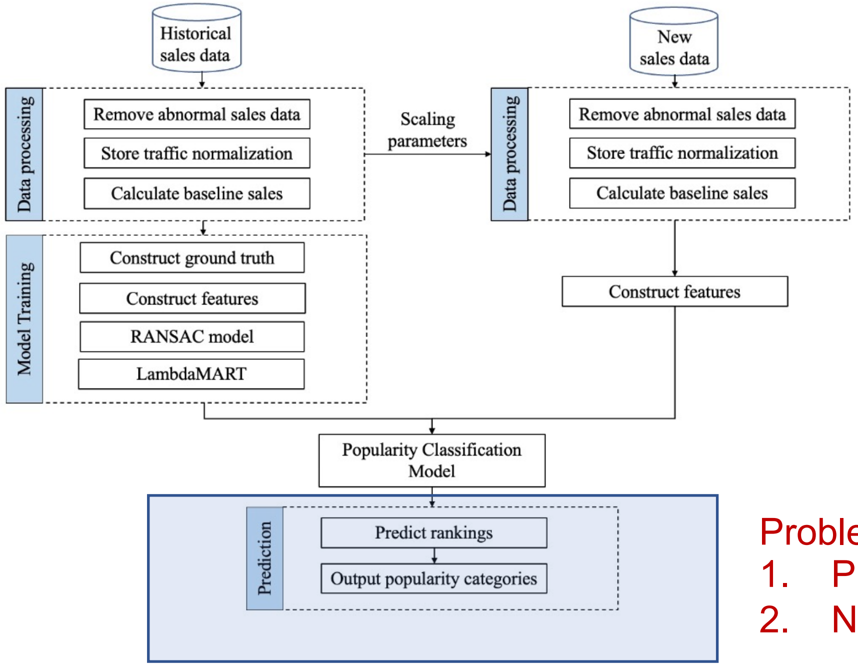
Popularity Classification Model

- Novel Indicator to Measure Popularity
- Novel Features
- Use ranking algorithm

Before: Feature => target => Ranking
Our model: Feature => Ranking

- Classification model that can automatically output the popularity rankings at time point t.
- **Informs Journal on Applied Analytics (Accept at 2023.11.04)**

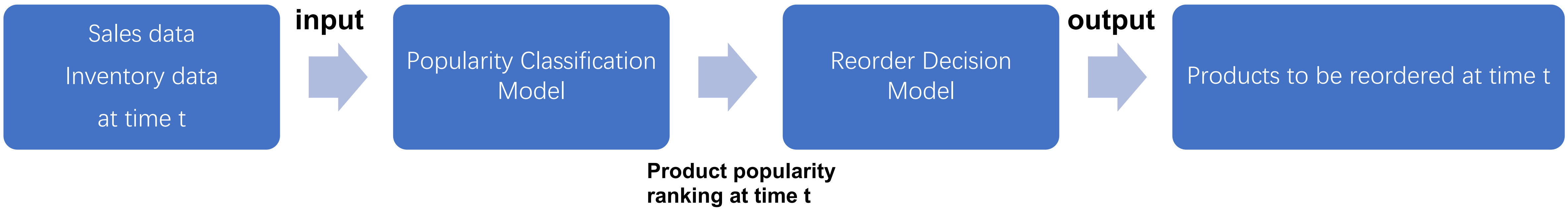
Figure 2 **Framework of the Popularity Classification Model**



Problem:
1. Prediction can be wrong
2. Naïve reorder policy

Reorder Strategy (working)

- Decision model that can automatically determine which products to reorder at time point t.



Reorder Decision Model

Prediction can be wrong.

When to believe it?

Single-product reorder point decision model

Model setting

Assumption:

1. Product category: fast-selling & slow-selling
2. Sales: sell in each store at respective speed given adequate inventory
3. Reorder : sell in n more stores.
4. Disposal value: zero
5. The parameter and distribution P(t) is known from historical data.

Notations

p: Product Pricing

c: Product cost

T: Maximum duration of the selling season

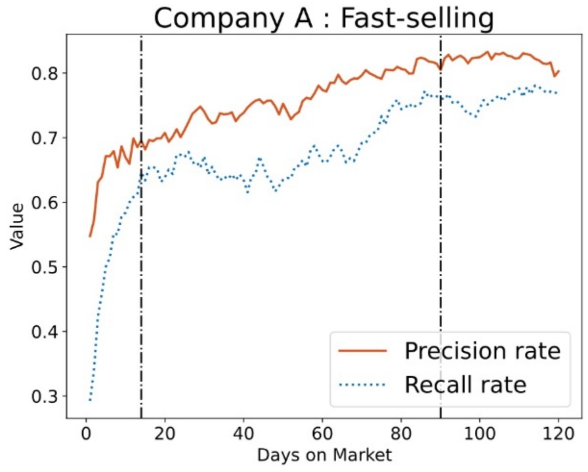
S_h: Average weekly store sales of fast-selling products.

S_l: Average weekly store sales of slow-selling products.

$P(t)$: Accuracy function of model identification

Profit Function

$$\begin{aligned} Profit(t) = & P(t)[n \cdot S_h \cdot (T-t) \cdot (p-c)] \\ & + [1-P(t)][n \cdot S_l \cdot (T-t) \cdot (p-c) \\ & - n \cdot (S_h - S_l) \cdot (T-t) \cdot c] \end{aligned} \tag{3}$$



(a)

P(t)

P(t) —— Linear, Piecewise linear, Concave

Simple case: $P(t) = a * t + b$

Profit function is convex in t.

$$Profit(t) = n \cdot [k_1 \cdot P(t) \cdot (T - t) + k_2 \cdot t] - k_2 * T$$
$$k_1 = p \cdot (S_h - S_l)$$
$$k_2 = c \cdot S_h - p \cdot S_l$$

$$t^* = [\frac{c}{p} + (\frac{c}{p} - 1) \cdot \frac{S_l}{S_h - S_l}] \cdot \frac{1}{2a} - \frac{b}{2a} + \frac{T}{2}$$
$$= \frac{k_2}{k_1} \cdot \frac{1}{2a} - \frac{b}{2a} + \frac{T}{2}$$

P(t) is concave

Theorem When the accuracy function P(t) satisfies the following properties, the profit function is also a concave function at this time.

(1) P(t) is a concave function

(2) The first derivative of P(t) is greater than zero

P(t) is piecewise linear with N segments

When risk is high: $\frac{k_2}{k_1} > \frac{a_{n-1} \cdot b_n - a_n \cdot b_{n-1}}{a_{n-1} - a_n}, n = 2, 3, \ldots, N$

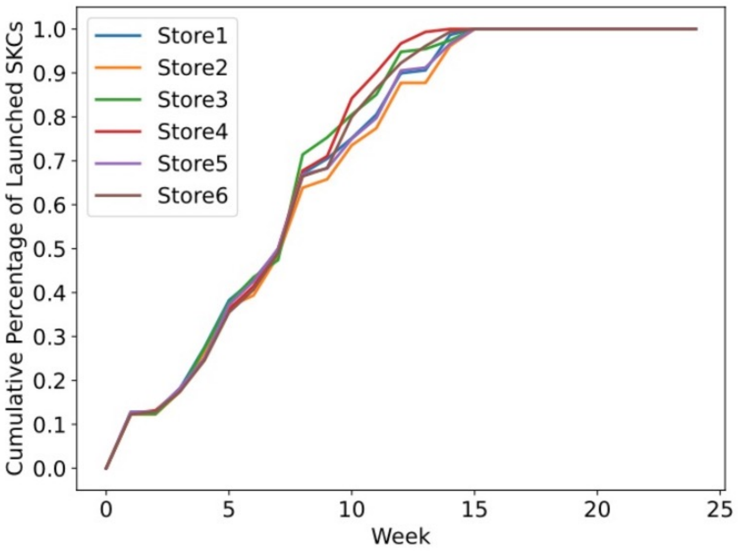
$$t^* = \begin{cases} 0, & \frac{k_2}{k_1} < b_1 - a_1 \cdot T \\ \alpha_1^*, & b_1 - a_1 \cdot T < \frac{k_2}{k_1} < b_1 - a_1 \cdot (T - 2\tau_0) \\ \alpha_2^*, & b_2 - a_2 \cdot (T - 2\tau_0) < \frac{k_2}{k_1} < b_2 - a_2 \cdot (T - 2\tau_1) \\ \cdots\cdots\cdots & \\ \alpha_n^*, & b_n - a_n \cdot (T - 2\tau_{n-2}) < \frac{k_2}{k_1} < b_n - a_n \cdot (T - 2\tau_{n-1}) \\ \cdots\cdots\cdots & \\ \alpha_N^*, & b_N - a_N \cdot (T - 2\tau_{N-2}) < \frac{k_2}{k_1} < b_N - a_N \cdot (T - 2\tau_{N-1}) \\ T, & b_N - a_N \cdot (T - 2\tau_{N-1}) < \frac{k_2}{k_1} \end{cases}$$

When risk is low: $\frac{k_2}{k_1} <= \frac{a_{n-1} \cdot b_n - a_n \cdot b_{n-1}}{a_{n-1} - a_n}, n = 2, 3, \ldots, N$

$$t^* = \begin{cases} 0, & \frac{k_2}{k_1} < b_1 - a_1 \cdot T \\ \alpha_1^*, & b_1 - a_1 \cdot T < \frac{k_2}{k_1} < b_1 - a_1 \cdot (T - 2\tau_0) \\ \tau_0, & b_1 - a_1 \cdot (T - 2\tau_0) < \frac{k_2}{k_1} < b_2 - a_2 \cdot (T - 2\tau_0) \\ \alpha_2^*, & b_2 - a_2 \cdot (T - 2\tau_0) < \frac{k_2}{k_1} < b_3 - a_3 \cdot (T - 2\tau_1) \\ \cdots\cdots\cdots & \\ \alpha_n^*, & b_n - a_n \cdot (T - 2\tau_{n-2}) < \frac{k_2}{k_1} < b_n - a_n \cdot (T - 2\tau_{n-1}) \\ \tau_{n-1}, & b_n - a_n \cdot (T - 2\tau_{n-1}) < \frac{k_2}{k_1} < b_{n+1} - a_{n+1} \cdot (T - 2\tau_{n-1}) \\ \cdots\cdots\cdots & \\ \alpha_N^*, & b_N - a_N \cdot (T - 2\tau_{N-2}) < \frac{k_2}{k_1} < 0 \end{cases}$$

Multiple Product Expansion Strategy (Heuristic)

- Products are introduced gradually over time
- 30% of products contribute 80% of total sales



Multi-Product Expansion Strategy

1. At the beginning of the sales season, set $t = 0$, $n = 0$, $\mathcal{F} = \emptyset$, $\gamma = 0$.

For $t < T$:

2. Update the model's output of the popular product set A_t at time t, and update the product launch ratio γ.

4. When $\frac{n}{0.3N} < \gamma$:

   For each product k in the popular product set A_t:

   Calculate the optimal expansion time point α_k^* for each product, and if $t \geq \alpha_k^*$, then expand product k.

   After expanding product k, update $\mathcal{F} \leftarrow \mathcal{F} \cup \{k\}$, $n \Leftarrow n + 1$.

   $t \Leftarrow t + 1$.

5. $t = T$, end.

Numerical Results

Table: Comparison of Sales Volume Between Two Expansion Strategies and Manual Methods

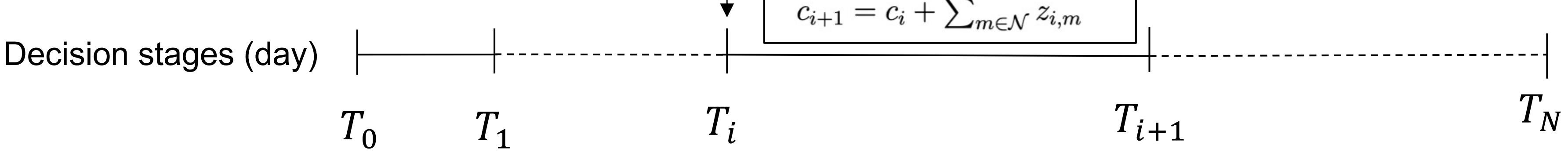
Method	Sales Volume Increase
Manual Identification Method	Baseline
Rule-based Expansion Strategy	5.90%
Multi-product Expansion Strategy	6.02%

Reorder Decision Model

MDP(Future work)

Exogenous state:
Popular product set $A_i := a_{i,m} \in 0, 1, m \in \mathcal{N}$

State transition
$c_{i+1} = c_i + \sum_{m \in \mathcal{N}} z_{i,m}$



Endogenous State:
Capacity $C_t \in [0, 0.3N]$

Action: choose products from A_i

$$Z_i(A_i) := \{z_{i,m} \in \{0,1\} \mid z_{i,m} = 0, \text{if } a_{i,m} = 0, \text{and } \textstyle\sum_{s=0}^{i} z_{s,m} \leq 1, \forall m \in \mathcal{N}\}$$

Reward:

$$r_i(A_i, c_i) = \sum_{m \in \mathcal{N}} z_{i,m} \dot{P}rofit(i, m)$$

$$\begin{aligned} Profit(i,m) = {} & P(\tau_{i,m})[n \cdot S_{hm} \cdot (T - i) \cdot (p_m - c_m)] \\ & + [1 - P(\tau_{i,m})][n \cdot S_l \cdot (T - t) \cdot (p - c) \\ & - n \cdot (S_{hm} - S_l) \cdot (T - i) \cdot c_m] \end{aligned}$$

Objective:

$$\max V(\emptyset, 0)$$

Recursive

$$V(A_i, c_i) = \max_{\pi} \{r_i(A_i, c_i) + \max\{V(A_{i+1}, c_{i+1})\}\}$$

Constraint:

$$c_i \leq 0.3 \cdot N, \quad \text{for all } i \in \{0, 1, \ldots, T\}$$